

A Robust Reflector Placement Framework for 60GHz mmWave Wireless Personal Area Networks

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Abstract—Multimedia streaming applications with stringent QoS requirements in 60GHz mmWave wireless personal area networks (WPANs) demand high rate and low latency data transfer as well as low service disruption. In this paper, we consider the problem of robust reflector placement in 60GHz WPANs. Reflectors relay traffic from transmitter devices to receiver devices facilitating i) the primary communication path for non-line-of-sight (NLOS) transceiver pairs, and ii) secondary (backup) communication path for line-of-sight (LOS) or NLOS transceiver pairs. We formulate the *robust minimum reflector placement* problem and the *robust maximum utility reflector placement* problem with the objective to minimize the number of reflectors deployed and maximize the network utility, respectively. Efficient algorithms are developed to solve both problems and have been shown to incur less service disruption in presence of moving subjects that may block the LOS paths in the environment.

I. INTRODUCTION

The *millimeter wave* (mmWave) band has attracted considerable commercial interests due to the advance in low-cost mmWave radio frequency integrated circuit design. The mmWave band provides 7GHz unlicensed spectrum resource at the center frequency of 60GHz¹, which would enable many high data rate applications like high definition streaming multimedia, high-speed kiosk data transfer and point-to-point terminal communication in data center [1]–[3], etc.

In contrast to many existing RF technologies such as 2.4GHz WiFi radios, mmWave radio has several unique physical characteristics. First, the propagation and attenuation loss are much more severe in the 60GHz band. It is shown that the free space path loss in 60GHz is more than 20dB larger than that in 5GHz. The oxygen absorption loss is as high as 5-30dB/km. Furthermore, the penetration loss is also much higher through typical building materials [1]. As a result, *line-of-sight* (LOS) path is the predominant path for signal transmission, while signals along the second-order and higher-order reflection paths are highly attenuated and often negligible. Second, to combat such significant signal degradation, directional antenna technology is essential in mmWave devices. By using directional antenna on both the transmitter and the receiver sides, mmWave radios can obtain

significant gain in the received signal strength, while incurring negligible interference to/from other radios [4]–[6]. In this paper, we consider an mmWave wireless personal area network (WPANs) equipped with directional antenna on all devices.

In addition to high bandwidth demands, multimedia applications in 60GHz WPANs also have stringent requirements on service disruption (defined as the frequency and duration of time that network connectivity is not available), which can occur due to changes in channel conditions such as LOS link blockage by moving subjects in the space. This motivates us to employ reflectors for two purposes. First, reflectors can be used to relay traffic from transmitters to receivers that do not have LOS connectivity. Second, reflectors can provide a secondary (2-hop) path in case of blockage on the primary (direct) path. We consider *active* reflector for the ease of control of reflection direction and signal boost.

In this paper, two reflector placement problems in 60GHz mmWave WPANs are investigated: *robust minimum reflector placement* (RMRP) that attempts to find the minimum number of reflectors and their best placements from a set of candidate locations with bandwidth and robustness constraints, and *robust maximum utility reflector placement* (RMURP) that aims to maximize network utility given a fixed number of reflectors. Two vertex-disjoint (except for the endpoints) paths (one called the primary path, and the other the secondary path) are provisioned between each pair of transmitter and receiver. Consequently, *seamless* switching to the secondary path is facilitated in event of channel degradation or blockage on the primary path avoiding service disruption. Robustness is characterized by the *D-norm* uncertainty model, which models tolerance to concurrent (worst-case) failures of a subset of primary paths [7]. Using linear relaxation and the duality theorem of linear programming, RMRP and RMURP are transformed to the mixed integer linear programming (MILP) and mixed integer non-linear programming (MINLP) problems. Two algorithms, *bisector search* and *Generalized Benders Decomposition*, are proposed for RMURP and are shown to have near optimal and optimal performance. Extensive simulations demonstrate the fault tolerance of the proposed reflector placement algorithms in significantly reducing the probability of service disruption.

The rest of this paper is organized as follows. The related literature work is analyzed in Section II. In Section III, we introduce the network model and the problem statements for robust reflector placement. The RMRP formulation is presented in Section IV, while RMURP is formulated in a similar way in Section V. Furthermore, we also discuss two proposed algorithms for RMURP in Section V. Performance

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¹57 - 64GHz in North America, and 59 - 66GHz in Europe and Japan

evaluation is presented in Section VI. Finally, we conclude this paper in Section VII.

II. RELATED WORK

Significant prior literature have been produced on different aspects of 60GHz radios, from CMOS circuit design to network protocol development. In this section, we summarize prior work on MAC design in mmWave WPANs.

A spatial time-division multiple access (STDMA) scheme was proposed for a realistic multi-Gbps mmWave WPAN in [8]. With the help of a heuristic scheduling algorithm, it is able to achieve significant throughput enhancement as much as 100% compared to conventional TDMA schedules. In [9], Cai *et al.* presented an efficient resource management framework based on the unique physical characteristics in a MC-DS-CDMA based mmWave networks. The authors also conducted extensive analysis of spatial multiplexing capacity in mmWave WPANs with directional antennae in [10], [11]. In [5], Madhow *et al.* conducted a probabilistic analysis of the interference in an mmWave network, as the result of uncoordinated transmission. It is concluded that an mmWave link can be abstracted as a “pseudo-wired link” with negligible interference when the beam width is 20 degree. Similar observations are made in [4], [12]. Therefore, the primary interference at the transmitter or receiver devices is the predominant source of contention. In [13], [14], to address the deafness problem induced by directionality, Gong *et al.* proposed a new directional CSMA/CA protocol for IEEE 802.15.3c 60GHz WPANs. With virtual carrier sensing, the central coordinator can distribute the network allocation vector (NAV) information, to avoid collisions among the devices occupying the same channel. The author also extended the work to a multiple-user scenario in [15]. A distributed scheduling protocol is proposed by coordinating mmWave mesh nodes in [6], and can achieve high resource utilization with time division multiplexing (TDM). However, none of the above work model or address reflector placement problems in mmWave WPANs with directional antenna.

There are also some existing work on repeater selection and relay operation scheduling in mmWave WPANs. Repeater selection was investigated in [16], with the objective to maximize data rate for each transmitter and receiver pair by determining the best link allocation. In [17], Lan *et al.* explored time slot scheduling for relay operations in the scenario of directional antenna on mmWave devices and formulated the throughput maximization problem as an integer programming problem. However, both schemes do not consider robustness in presence of uncertain link blockage.

To the best of our knowledge, we are the first to explore robust reflector placement in 60GHz WPANs. Some preliminary results are presented in [18].

III. NETWORK MODEL AND PROBLEM STATEMENT

Consider an mmWave network consisting of a set of L *logical mmWave links* (simplified as *logical links*), each link $i \in L$ is associated with a source device (transmitter) s_i , a

destination device (receiver) d_i , and a traffic demand r_i bps. *mmWave reflector devices* (simplified as *reflectors*) equipped with steerable antennas can relay data between the transmitters and receivers. The reflectors can be placed at a set of K candidate locations. We further consider a set of O obstacles in the environment with known locations.

A. Geometric Model for Link Connectivity

In this section, we introduce a geometric model to characterize link connectivity in 60GHz mmWave WPANs. LOS transmissions are feasible between a transmitter and a receiver if and only if the direct path between them is unobstructed and their distance is less than a threshold d .

To bound the end-to-end latency, at most two-hop paths (involving one reflector) are allowed between any transmitter-receiver pair². Thus, the connectivity of any logical link is determined by the visibility regions of its end points defined as follows:

Definition 1: (Visibility region) Given a 2-D plane of interest, any two points (a, b) are visible to each other if the line segment between them does not intersect with any obstacles and the distance of a and b is no more than d . The *visibility region* $V(a)$ of a point a in the plane is the bounded shape consisting of all unobstructed points no more than distance d from a .

Consider transmitter and receiver a and b , the (binary) connectivity of logical link (a, b) is thus characterized by:

$$\lambda(a, b) = \begin{cases} 1, & \text{iff } V(a) \cap V(b) \neq \emptyset, \\ 0, & \text{otherwise} \end{cases}, \quad (1)$$

If $\lambda(a, b) = 1$, logical link (a, b) is *feasible* (directly or via a reflector in a and b 's overlapping visibility region); otherwise, it is *infeasible*. For the rest of the paper, we only consider the set of feasible logical links given by:

$$\Omega = \{i \mid \lambda(s_i, d_i) = 1, \forall i \in L\}, \quad (2)$$

where s_i, d_i are the transmitter and receiver of the i -th logical link respectively.

Fig. 1 illustrates the notion of visibility region. There are two mmWave end devices, DEV1 and DEV2. The shadowed area corresponds to the overlapped visibility region between DEV1 and DEV2, which is the candidate region for reflector placement for DEV1 and DEV2. The existence of overlapped visibility region is the necessary and sufficient condition of the connectivity of logical links, in both LOS and *non-LOS* (NLOS) cases.

The relationship between feasible logical links and candidate reflector locations can be modeled as an undirected bipartite graph $G(\Omega, \mathcal{K}, E)$, where $\mathcal{K} = 1, \dots, K$ is the set of candidate reflector locations. An edge $e = (l, k)$ exists between a logical link $l \in \Omega$ and a candidate reflector location $k \in \mathcal{K}$ iff k is in $V(s_l) \cap V(d_l)$, where s_l and d_l are the end points of l . Thus, the set of feasible logical links that can use

²The proposed solutions can be extended to cases where longer paths are allowed.

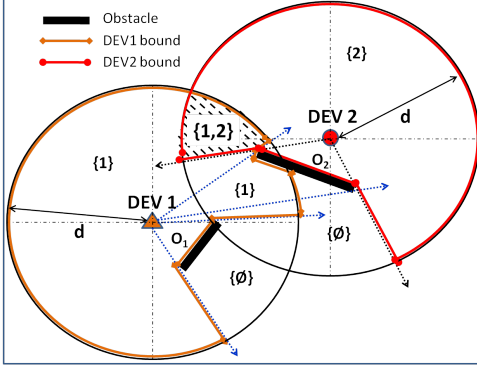


Fig. 1: Visibility region and overlapped visibility region.

a reflector placed at the candidate location k as relay is given by:

$$\Omega_k = \{i \mid k \in V(s_i) \cap V(d_i), \forall i \in \Omega\}, \forall k \in \mathcal{K}. \quad (3)$$

B. Needs for robustness

Reflectors serve two purposes: *i*) providing the primary communication path for NLOS logical links; and *ii*) providing secondary (backup) communication path for LOS or NLOS logical links. Provisioning of secondary paths reduces service disruption when the primary path is obstructed.

To see the impact of secondary paths, we conduct a simple simulation study. Consider a home-network environment in Fig. 2(a), where there is a LOS logical link l and a dedicated reflector at a fixed location. In the robust setting, one reflector is used to provide a secondary communication path for l . Inside the room, there are M moving human subjects modeled as a circle with a radius of 0.3 meters. We adopt the random walk model [19], where in each step, a person moves 0.3 meters with the direction randomly chosen from the set $\{-90^\circ, -45^\circ, 0^\circ, 45^\circ, 90^\circ\}$. Without reflectors, the communication between TX and RX is disrupted when a person blocks the direct LOS path. With reflectors, an outage occurs only when both the primary (LOS) path and the secondary path (via the reflector) are blocked. Fig. 2(b) and Fig. 2(c) show the percentage of blockage time and the mean, max and min duration with different number of human subjects, respectively.

As shown in Fig. 2 (b)(c), when the number of moving subjects increases, the percentage of link blockage and blockage duration increase with and without the secondary path. However, the use of backup path reduces both the blocking probability and the duration of each outage. This translates to better quality of service (QoS) at the application layer.

C. Problem Statement

Reflector placement concerns with the selection of reflectors among a finite set of candidate locations to optimize for certain network utilities. We consider two variations of the problem.

Definition 2: (Robust Minimum Reflector Placement (RMRP) problem) Given an mmWave network with a set of feasible logical links with fixed traffic demands, find the

minimum number of reflectors and their locations among candidate location set \mathcal{K} that satisfy connectivity, bandwidth, and robustness constraints.

Definition 3: (Robust Maximum Utility Reflector Placement (RMURP) problem) Given an mmWave network with a set of feasible logical links with base traffic demands, find the placement of at most m reflectors among candidate location set \mathcal{K} such that the ratio of the achievable rates over the base rate is maximized subject to robustness constraints.

We restrict relay of data to reflectors only. In the robust formulation, for each feasible logical link, two vertex-disjoint (except for the endpoints) communication paths are provisioned, one as *primary path*, and the other as *secondary path*. Both the primary and secondary paths between mmWave transmitters and receivers cannot be more than 2-hops. If a reflector serves as relay for more than one logical links, *time division medium access* (TDMA) scheduling is adopted. The interference among concurrent transmissions is assumed to negligible due to the high directionality of mmWave communications, which is consistent with the measurement results reported in [6]. Therefore, the main sources of contention arise from the half-duplex constraint and multiplexing at the reflector nodes.

IV. ROBUST MINIMUM REFLECTOR PLACEMENT (RMRP)

In this section, we present the analytical form of the RMRP problem. The following notations are used:

- **Primary indicator:** $x_{ik} = 1$ if reflector k is selected by logical link i as its primary path reflector; otherwise, $x_{ik} = 0$;
- **Secondary indicator:** $y_{ik} = 1$ if reflector k is selected by logical link i as its secondary path reflector; otherwise, $y_{ik} = 0$;
- **Selection indicator:** $z_k = 1$ if reflector k is selected by at least one logical link; otherwise, $z_k = 0$.
- **NLOS indicator:** $\eta_i = 1$ if logical link i does not have a LOS path; otherwise, $\eta_i = 0$.

If a logical link i has a LOS path, no reflector is needed for the primary path; otherwise, one reflector should be selected for the primary path, which should satisfy the following condition:

$$\sum_{k=1}^K x_{ik} = \eta_i, \forall i \in \Omega. \quad (4)$$

On the other hand, at least one reflector is needed to facilitate the secondary path, that is,

$$\sum_{k=1}^K y_{ik} = 1, \forall i \in \Omega. \quad (5)$$

In addition, a reflector cannot be used for the primary path and the secondary path simultaneously. Therefore, we have:

$$x_{ik} + y_{ik} \leq 1, \forall i \in \Omega, \forall k. \quad (6)$$

As mentioned before, for simplicity, we assume each reflector has only one half-duplex transceiver. Therefore, the

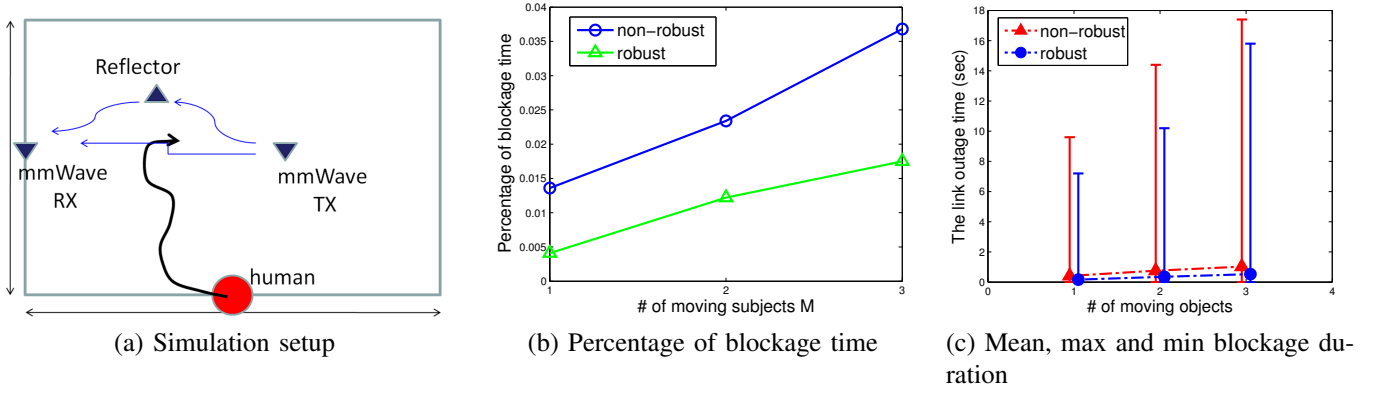


Fig. 2: Link blockage with and without reflectors due to moving human subjects in a $10\text{m} \times 10\text{m}$ room with a fixed mmWave TX/RX and a dedicated reflector, M human subjects moving randomly inside.

transmission time of relaying a unit data of logical link i via reflector k is

$$\tau_{ik} = \frac{1}{R_{s_i,k}} + \frac{1}{R_{k,d_i}}, \quad (7)$$

where $R_{s_i,k}$, R_{k,d_i} are the mmWave data bandwidth between the source and the reflector, and between the reflector and the destination, respectively. For the AWGN channel, they can be modeled as:

$$R_{s_i,k} = \begin{cases} W \log \left[1 + \frac{P_t G_t G_r}{P_n D(s_i, k)^\gamma} \right] & (\text{when } D(s_i, k) \leq \Theta) \\ 0 & (\text{when } D(s_i, k) > \Theta), \end{cases} \quad (8)$$

and

$$R_{k,d_i} = \begin{cases} W \log \left[1 + \frac{P_t G_t G_r}{P_n D(k, d_i)^\gamma} \right] & (\text{when } D(k, d_i) \leq \Theta) \\ 0 & (\text{when } D(k, d_i) > \Theta), \end{cases} \quad (9)$$

respectively, where W is the channel bandwidth in Hz, P_t is the transmission power, P_n is the noise floor level, G_t , G_r are transmitter and receiver antenna gains, γ is the large-scale path loss index, $D(a, b)$ is the distance from a to b , and Θ is a constant threshold on communications radius.

For a reflector k , the TDMA scheduling for the associated logical links should satisfy:

$$\sum_{i \in \Omega_k} \eta_i x_{ik} r_i \tau_{ik} + g_k(\mathbf{y}_k, \mathbf{r}) \leq z_k, \forall k, \quad (10)$$

where r_i is the traffic demand of logical link i , τ_{ik} is the unit data relay time of i via reflector k . The first term on the left side represents the percentage of reflector capacity occupied by all the logical links using this reflector as their primary path reflector. The second term represents the *protection function* for the set of logical links using this reflector as their secondary path reflector. A protection function $g_k(\cdot)$ measures the robustness of an mmWave WPAN. Its meaning will be further explained in Section IV-A.

To this end, the RMRP problem can be formally stated as:

$$\begin{aligned} & \underset{\mathbf{x}, \mathbf{y}, \mathbf{z}}{\text{minimize}} && \sum_k z_k \\ & \text{subject to} && \text{Constraints (4) – (10)} \\ & \text{variables} && x_{ik}, y_{ik}, z_k \in \{0, 1\}, \forall i \in \Omega, k = 1, \dots, K \end{aligned} \quad (11)$$

A. Reformulation under the D -norm uncertainty model

Several uncertainty models have been proposed in literature, including *General Polyhedron*, *D-norm*, *Ellipsoid*, etc [7]. In this paper, we adopt the D -norm uncertainty model that is characterized by the following protection function:

$$g_k(\mathbf{y}_k, \mathbf{r}) = \max_{S_k: S_k \subseteq \Omega_k, |S_k| = \Gamma_k} \sum_{i \in S_k} y_{ik} r_i \tau_{ik}. \quad (12)$$

Under the D -norm uncertainty model, among the set of logical links Ω_k that can relay through reflector k , at most Γ_k links will be blocked simultaneously on the primary path and consequently transmit on their secondary path via reflector k . The maximization gives the worst case traffic loads induced on the reflector.

Two special cases are of particular interest. If $\Gamma_k = |\Omega_k|$, then $g_k(\mathbf{y}_k, \mathbf{r}) = \sum_{i \in \Omega_k} y_{ik} r_i \tau_{ik}$. This means all logical links in Ω_k fail simultaneously. This corresponds to the maximum robustness. In this case, more reflectors may be needed. At the other extreme, if $\Gamma_k = 0$, no logical link is blocked. Fewer reflectors are in use. However, there is little fault tolerance in the resulting reflector placement. Denote $\rho \equiv \Gamma_k / |\Omega_k|$ the *robustness index* ρ is a parameter to tradeoff between robustness and resource usage. .

Under the above D -norm uncertainty model, (10) can be rewritten as:

$$\sum_{i \in \Omega_k} \eta_i x_{ik} r_i \tau_{ik} + \max_{S_k: S_k \subseteq \Omega_k, |S_k| = \Gamma_k} \sum_{i \in S_k} y_{ik} r_i \tau_{ik} \leq z_k, \forall k, \quad (13)$$

(13) is not directly tractable since it involves an inner-optimization in the protection function. The protection function can be reformulated an integer linear programming prob-

lem as follows [7]:

$$\begin{aligned} & \max_{\{0 \leq s_{ik} \leq 1\} \forall i \in \Omega_k} \sum_{i \in \Omega_k} y_{ik} r_i \tau_{ik} s_{ik}, \\ & \text{s.t.} \quad \sum_{i \in \Omega_k} s_{ik} \leq \Gamma_k, \\ & \quad s_{ik} \in \{0, 1\}, \forall i \in \Omega_k \end{aligned} \quad (14)$$

Consider a linear relaxation of the above problem where $s_{ik} \in [0, 1]$. Due to the linearity of the constraints, the optimal solution occurs at the vertices of the feasibility region. Hence the optimal solution s_{ik}^* must be either 0 or 1, as there is no gap between the integer linear programming and the linear programming solutions.

Taking the dual of the linear programming problem (14), we have:

$$\begin{aligned} & \min_{\{p_{ik} \geq 0\} \forall i \in \Omega_k, q_k \geq 0} q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik}, \\ & \text{s.t.} \quad q_k + p_{ik} \geq y_{ik} r_i \tau_{ik}, \end{aligned} \quad (15)$$

Substituting (15) into (11), we can obtain the equivalent formulation of the RMRP problem as a *mixed integer linear programming* (MILP) problem as follows:

$$\begin{aligned} & \min_{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{p}, \mathbf{q}} \sum_k z_k \\ & \text{s.t.} \quad \sum_{i \in \Omega_k} \eta_i x_{ik} r_i \tau_{ik} + q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik} \leq z_k, \forall k \\ & \quad q_k + p_{ik} \geq y_{ik} r_i \tau_{ik}, \forall i \in \Omega_k, \forall k \\ & \quad \text{Constraints (4)(5)(6)(7)(8)(9)} \\ & \quad \text{variables} \quad x_{ik}, y_{ik}, z_k \in \{0, 1\}, p_{ik} \geq 0, q_k \geq 0 \end{aligned} \quad (16)$$

B. Hardness of RMRP

We prove in Appendix A that RMRP is NP-hard.

The MILP problem in (16) can be solved by any MILP solver. In our implementation, we adopt the MILP solver of the IBM optimization tool – CPLEX [20].

V. ROBUST MAXIMUM UTILITY REFLECTOR PLACEMENT (RMURP)

In contrast to RMRP, which tries to minimize the number of reflectors, RMURP aims to maximize the total utility of an mmWave WPAN given a fixed number of reflectors.

Let r_i be the base traffic demand on logical link i . We allow r_i be scaled up/down according to network constraints. That is, the actual data rate supported is αr_i , where α is a scaling parameter. This formulation is particularly relevant for transferring multimedia content that allows adaptive encoding. The objective of RMURP is henceforth to maximize the total utility U of the network, given by $U = \sum_i \alpha r_i$.

The constraints of RMURP are similar to those of RMRP, except that the TDMA schedulability constraint (10) now becomes

$$\sum_{i \in \Omega_k} \eta_i x_{ik} \alpha r_i \tau_{ik} + g_k(\mathbf{y}_k, \alpha \mathbf{r}) \leq z_k, \forall k, \quad (17)$$

and additional cardinality constraints need to be included:

$$\sum_k z_k \leq m, \quad (18)$$

where m is the maximum number of reflectors to be used.

To this end, the RMURP can be formalized as:

$$\begin{aligned} & \text{maximize}_{\mathbf{x}, \mathbf{y}, \mathbf{z}, \alpha} \quad \sum_i \alpha r_i \\ & \text{subject to} \quad \text{Constraints (4)(5)(6)(7)(8)(9)(18)(17)} \\ & \quad \text{variables} \quad x_{ik}, y_{ik}, z_k \in \{0, 1\}, \alpha \geq 0, \forall i, k \end{aligned} \quad (19)$$

A. Reformulation under D-norm Uncertainty Model

Again we can apply the D -norm uncertainty model (see Section IV-A) to the RMURP problem of (19). This will transform (19) into a *mixed integer non-linear programming problem* (MINLP) as follows:

$$\begin{aligned} & \text{Maximize}_{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{p}, \mathbf{q}, \alpha} \quad \sum_i \alpha r_i \\ & \text{subject to} \quad \sum_{i \in \Omega_k} \eta_i x_{ik} \alpha r_i \tau_{ik} + q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik} \leq z_k, \forall k \\ & \quad q_k + p_{ik} \geq y_{ik} \alpha r_i \tau_{ik}, \forall i \in \Omega_k, \forall k \\ & \quad \text{Constraints (4)(5)(6)(7)(8)(9)(18)} \\ & \quad \text{variables} \quad x_{ik}, y_{ik}, z_k \in \{0, 1\}, p_{ik} \geq 0, q_k \geq 0, \alpha \geq 0 \end{aligned} \quad (20)$$

B. Hardness of RMURP

We prove in Appendix B that RMURP is NP-hard. The inclusion of variable α renders RMURP an MINLP. Specialized algorithms need to be designed.

Next, we propose two algorithms to solve the RMURP. The first algorithm is based on *Bisection Search*, which is shown to be fast but does not guarantee optimality. The second algorithm is based on the *Generalized Benders' Decomposition* (GBD) technique [21], [22], which is proven to converge to optimal solutions but has higher computation complexity.

C. Bisection Search

Bisection Search is a heuristic method for finding roots of an equation. It iteratively bisects an interval and then selects the subinterval where a root must reside for the next iteration, until some termination condition is met. It is guaranteed to converge to a root of $f(\cdot)$ if and only if: f is a continuous function on the interval $[A, B]$, and $f(A)$ and $f(B)$ have opposite signs.

In (20), if α is given, RMURP becomes a MILP, which can be solved by CPLEX. The key is thus to determine the value of α . When α is large, RMURP is infeasible. When $\alpha = 0$, RMURP is always feasible. More generally, if $\alpha_1 > \alpha_2$, RMURP is infeasible when $\alpha = \alpha_1$, then, RMURP is feasible when $\alpha = \alpha_2$. Treating feasibility and infeasibility as opposite signs, we apply the *Bisection Search* principle to decide the range of α iteratively until it is smaller than a threshold $2 \times TOL$. Starting from an initial interval $[A, B]$, where $\alpha = A$ renders RMURP feasible and $\alpha = B$ renders RMURP infeasible, we substitute A or B with $\frac{A+B}{2}$ depending on the feasibility of RMURP under $\alpha = \frac{A+B}{2}$. The “monotonicity” in the feasibility of RMURP with respect to α makes the *Bisection Search* converge fast but the optimality of the final results depends on TOL .

The *Bisection Search* based algorithm is summarized in Algorithm 1.

Algorithm 1: Bisection Search

Input : E2e data rate demands of every feasible mmWave logical link $r_i, \forall i$; error tolerance TOL (which serves as the iteration termination condition); and the up-to-date known range for α : $[A, B]$

Output: Maximum network utility U_T and reflector selection variables for every feasible mmWave links $\mathbf{x}, \mathbf{y}, \mathbf{z}$

```

begin
  Set  $n = 1$ ;
  while  $n \leq \max N$  do
     $C \leftarrow \frac{(A+B)}{2}$ ;
    Solve the MILP problem  $RMURP(C)$ .
    if  $RMURP(C)$  is feasible then
       $A \leftarrow C$ ;
      Obtain the solutions of  $RMURP(C)$ :  $\mathbf{x}^{(n)}, \mathbf{y}^{(n)}, \mathbf{z}^{(n)}$ .
    else
       $B \leftarrow C$ ;
    end
     $n \leftarrow n + 1$ ;
    if  $\frac{(B-A)}{2} \leq TOL$  then
      The best  $\alpha$  found,  $\alpha_{best} \leftarrow A$ ;
      Return  $U_T, \mathbf{x}^{(n)}, \mathbf{y}^{(n)}, \mathbf{z}^{(n)}$ .
    end
  end
end

```

D. Generalized Benders' Decomposition (GBD)

GBD is an iterative method for solving MINLP problems. The principle of the GBD algorithm is to decompose the original MINLP problem into a *primal problem* and a *master problem*, and then solve them iteratively. The primal problem corresponds to the original problem with fixed binary variables. Solving the primal problem provides a lower bound, and Lagrange multipliers corresponding to the constraints. The master problem is derived through nonlinear duality theory using the Lagrange multipliers obtained from the primal problem. Solving the master problem provides an upper bound, and binary variables that can be used for the primal problem in the next iteration. It is proven to converge to the optimum [21].

Primal Problem: Let $\Lambda := (\mathbf{x}, \mathbf{y}, \mathbf{z})$ represent the set of binary variables, $\hat{\Lambda} := (\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ indicates the binary variables with specific values in $\{0, 1\}$. The primal problem $\mathcal{P}(\hat{\Lambda})$ of RMURP problem (20) is obtained by fixing all the binary variables to $\hat{\Lambda}$ as follows:

$$\begin{aligned}
 f(\hat{\Lambda}) = & \underset{\mathbf{p}, \mathbf{q}, \alpha}{\text{maximize}} \quad \sum_i \alpha r_i \\
 \text{subject to} \quad & \sum_{i \in \Omega_k} \eta_i \hat{x}_{ik} \alpha r_i \tau_{ik} + q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik} \leq \hat{z}_k, \forall k \\
 & q_k + p_{ik} \geq \hat{y}_{ik} \alpha r_i \tau_{ik}, \forall i \in \Omega_k, \forall k \\
 \text{variables} \quad & \mathbf{p} \succeq 0, \mathbf{q} \succeq 0, \alpha \geq 0
 \end{aligned} \tag{21}$$

Problem (21) is a linear programming problem, which can be solved by any linear programming solver. Since the optimal solution of $\mathcal{P}(\hat{\Lambda})$ is also a feasible solution to (20), the optimal value $f(\hat{\Lambda})$ provides a lower bound to the RMURP. It is also clear that, not all the choices of given binary variables can lead

to a feasible primal problem. We need to treat it differently depending on whether the primal problem is feasible or not:

- **Feasible Primal:**

If the primal problem is feasible, let

$$\begin{aligned}
 T_k(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha) &= \hat{z}_k - \left(\sum_{i \in \Omega_k} \eta_i \hat{x}_{ik} \alpha r_i \tau_{ik} + q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik} \right), \forall k, \\
 g_{ik}(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha) &= q_k + p_{ik} - \hat{y}_{ik} \alpha r_i \tau_{ik}, \forall i \in \Omega_k, \forall k.
 \end{aligned} \tag{22}$$

Then, we can compute the *partial Lagrangian* function for the primal problem as follows:

$$L(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha, \lambda, \nu) = \sum_i \alpha r_i + \sum_k \lambda_k T_k + \sum_k \sum_i \nu_{ik} g_{ik}, \tag{23}$$

where $\lambda_k, \nu_{ik} \geq 0, \forall i \in \Omega_k, \forall k$ are the *Lagrange multipliers*.

Thus, the *Lagrange dual* problem of $\mathcal{P}(\hat{\Lambda})$ can be stated as:

$$\min_{\lambda, \nu} \max_{\mathbf{p}, \mathbf{q}, \alpha} L(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha, \lambda, \nu). \tag{24}$$

Since the problem is convex and has linearity constraints, the duality gap is 0. Thus, solving the Lagrange dual problem would give the optimal solution for $\mathcal{P}(\hat{\Lambda})$.

- **Infeasible Primal:**

If the primal problem is infeasible, we first define a set Δ as:

$$\Delta = \{\hat{\Lambda} | T_k \geq 0, g_{ik} \geq 0, \forall i \in \Omega_k, \forall k, \text{ for some } \mathbf{p}, \mathbf{q}, \alpha\}, \tag{25}$$

and consider the following feasibility-checking problem $\mathcal{F}(\hat{\Lambda})$:

$$\begin{cases}
 \underset{\mathbf{p}, \mathbf{q}, \alpha}{\text{minimize}} & \delta \\
 \text{subject to} & \sum_{i \in \Omega_k} \eta_i \hat{x}_{ik} \alpha r_i \tau_{ik} + q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik} - \hat{z}_k \leq \delta, \forall k \\
 & \hat{y}_{ik} \alpha r_i \tau_{ik} - q_k - p_{ik} \leq \delta, \forall i \in \Omega_k, \forall k \\
 \text{variables} & \mathbf{p} \succeq 0, \mathbf{q} \succeq 0, \alpha \geq 0, \delta \geq 0
 \end{cases} \tag{26}$$

It is straightforward to see that, for any given $\hat{\Lambda}$, $\mathcal{P}(\hat{\Lambda})$ is infeasible if and only if $\mathcal{F}(\hat{\Lambda})$ has a positive optimal value $\delta^* > 0$.

The *Lagrangian* function for $\mathcal{F}(\hat{\Lambda})$ can be presented as:

$$\begin{aligned}
 G(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha, \mu, \sigma) &= \sum_k \mu_k \left(\sum_{i \in \Omega_k} \eta_i \hat{x}_{ik} \alpha r_i \tau_{ik} + q_k \Gamma_k + \sum_{i \in \Omega_k} p_{ik} - \hat{z}_k \right) \\
 &\quad + \sum_k \sum_{i \in \Omega_k} \sigma_{ik} (\hat{y}_{ik} \alpha r_i \tau_{ik} - q_k - p_{ik}), \\
 &\quad \forall (\mu_k, \sigma_{ik}) \in \mathcal{U}
 \end{aligned} \tag{27}$$

where μ_k, σ_{ik} are Lagrange multipliers and $\mathcal{U} = \{(\mu_k, \sigma_{ik}) | \mu_k, \sigma_{ik} \geq 0, \sum_k (\mu_k + \sum_{i \in \Omega_k} \sigma_{ik}) = 1, \forall i \in \Omega_k, \forall k\}$.

The Lagrangian dual of $\mathcal{F}(\hat{\Lambda})$ becomes:

$$\max_{\mu, \sigma} \min_{\mathbf{p}, \mathbf{q}, \alpha} G(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha, \mu, \sigma). \tag{28}$$

Therefore, for any $\hat{\Lambda} \in \Delta$, it can be characterized by the inequality constraint:

$$0 \geq \min_{\mathbf{p}, \mathbf{q}} G(\hat{\Lambda}, \mathbf{p}, \mathbf{q}, \alpha, \mu, \sigma). \tag{29}$$

Master Problem: The original problem in (20) can be written as:

$$\begin{aligned}
\max_{\Lambda} \sum_i \alpha r_i &= \max_{\Lambda \in \Delta} f(\Lambda) \\
&= \max_{\Lambda \in \Delta} \left[\min_{\lambda, \nu} \max_{\mathbf{p}, \mathbf{q}, \alpha} L(\Lambda, \mathbf{p}, \mathbf{q}, \alpha, \lambda, \nu) \right] \\
&= \max_{\beta} \quad \text{s.t. } \beta \leq \max_{\mathbf{p}, \mathbf{q}, \alpha} L(\Lambda, \mathbf{p}, \mathbf{q}, \alpha, \lambda, \nu), \forall \lambda, \nu \succeq 0 \\
&\quad \Lambda \in \{0, 1\} \cap \Delta,
\end{aligned} \tag{30}$$

where the second equality is due to (24) because of the zero duality gap. Incorporating (29) into (30), we finally obtain the master problem $\mathcal{M}(\mathbf{p}, \mathbf{q}, \alpha, \lambda, \nu, \mu, \sigma)$ as:

$$\mathcal{M}(\cdot) \left\{ \begin{array}{l} \max_{\Lambda} \quad \beta \\ \text{s.t.} \quad \beta \leq \max_{\mathbf{p}, \mathbf{q}, \alpha} L(\Lambda, \mathbf{p}, \mathbf{q}, \alpha, \lambda, \nu), \forall \lambda, \nu \succeq 0 \\ 0 \geq \min_{\mathbf{p}, \mathbf{q}, \alpha} G(\Lambda, \mathbf{p}, \mathbf{q}, \alpha, \mu, \sigma), \forall (\mu, \sigma) \in \mathcal{U} \\ \text{Constraints (4)(5)(6)(7)(8)(9)(18)} \\ \Lambda \in \{0, 1\}, \beta \geq 0 \end{array} \right. \tag{31}$$

Note that, the master problem has two inner optimization problems as its constraints, which need to be considered for all λ, ν and μ, σ . This implies that the master problem has a very large number of constraints. In order to obtain a solvable mixed-integer linear programming problem, we employ the following relaxation for the master problem at iteration n as suggested by [21]:

$$\begin{aligned}
\beta &\leq L(\Lambda^n, \mathbf{p}^n, \mathbf{q}^n, \alpha^n, \lambda^n, \nu^n) + \nabla_{\Lambda} L(\cdot)(\Lambda - \Lambda^n), \forall n \in \mathcal{P}^k \\
0 &\geq G(\Lambda^n, \mathbf{p}^n, \mathbf{q}^n, \alpha^n, \mu^n, \sigma^n) + \nabla_{\Lambda} G(\cdot)(\Lambda - \Lambda^n), \forall n \in \mathcal{F}^k,
\end{aligned} \tag{32}$$

where \mathcal{P}^k and \mathcal{F}^k are the sets of feasible and infeasible primal problems solved up to iteration k , respectively.

The relaxed problem provides the upper bound of the original master problem and also provide the value of binary variables for the primal problem in the next iteration.

The *GBD* algorithm is summarized in Algorithm 2.

It has been proved in [21] that, the solutions to discrete variables $\Lambda^1, \dots, \Lambda^k$ do not repeat. Therefore, due to the finiteness of the discrete variable set, *GBD* algorithm converges within a finite number of iterations. When it converges, the lower bound is equal to the upper bound. Thus, optimality is achieved.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the reflector placement solutions using simulations. In the simulations, an mmWave home network is deployed in a 10m×10m room, where N mmWave devices and O obstacles are uniformly placed. The reflectors can be placed at any grid point in a grid separated by distance d_0 . The transmission radii of all mmWave end devices and reflectors are set to 6 meters. The (base) traffic demand r_i of each logical link i is chosen as $\frac{1}{3}$ of the AWGN Shannon channel capacity of the slowest LOS path. The PHY parameters are given in Table I. In all experiments, $d_0 = 2m$, $O = 10$.

Algorithm 2: Generalized Benders' Decomposition

Input : Base logical data rate r_i for each feasible logical link i
Output: Maximum network utility adapt factor α and reflector selection variables for every feasible logical link $\Lambda = (\mathbf{x}, \mathbf{y}, \mathbf{z})$ and \mathbf{p}, \mathbf{q}

```

begin
  set  $n = 1$  and choose  $\Lambda \in \{0, 1\}$ ,
   $LB^0 \leftarrow -\infty, UB^0 \leftarrow \infty, \mathcal{P}^0 \leftarrow \emptyset, \mathcal{F}^0 \leftarrow \emptyset$ .
  while  $LB^{n-1} \leq UB^{n-1}$  do
    if the primal problem is feasible then
      Solve the primal problem  $\mathcal{P}(\Lambda^n)$  to obtain optimal
      solution  $\mathbf{p}^n, \mathbf{q}^n, \alpha^n$ 
      and Lagrangian multipliers  $\lambda^n, \nu^n$ ;
       $\mathcal{P}^n \leftarrow \mathcal{P}^{n-1} \cup \{n\}, \mathcal{F}^n \leftarrow \mathcal{F}^{n-1}$ ;
       $LB^n \leftarrow \max(LB^{n-1}, f(\Lambda^n))$ ;
      if  $LB^n = f(\Lambda^n)$  then
        |  $(\Lambda^*, \tilde{\mathbf{p}}^*, \tilde{\mathbf{q}}^*, \tilde{\alpha}^*) \leftarrow (\Lambda^n, \tilde{\mathbf{p}}^n, \tilde{\mathbf{q}}^n, \tilde{\alpha}^n)$ ;
      end
    else if the primal problem is infeasible then
      Solve the feasibility-check problem  $\mathcal{F}(\hat{\Lambda})$  to obtain the
      optimal solution  $\mathbf{p}^n, \mathbf{q}^n, \alpha^n$  and Lagrangian multipliers
       $\mu^n, \sigma^n$ ;
       $\mathcal{P}^n \leftarrow \mathcal{P}^n, \mathcal{F}^n \leftarrow \mathcal{F}^{n-1} \cup \{n\}$ ;
    end
    Solve the master problem  $\mathcal{M}(\mathbf{p}^n, \mathbf{q}^n, \alpha^n, \lambda^n, \nu^n, \mu^n, \sigma^n)$ 
    and obtain the optimal solution  $\Lambda^{n+1}$  and  $\beta^n$ ;
     $UB^n \leftarrow \beta^n, n \leftarrow n + 1$ ;
  end
  return  $\Lambda^*, \mathbf{p}^*, \mathbf{q}^*, \alpha^*$ .
end

```

TABLE I: PHY Parameters

PHY parameters	Values
Channel	AWGN with gain 1
Path Loss	free space, exponent 2
Transmission power	20mW (13dBm)
Noise floor	-100dBm

A. RMRP Performance

In this section, we examine the performance of RMRP under different configurations by varying the number of mmWave logical links (N), the number of moving subjects (M) and the robustness index (ρ).

Fig. 3(a) shows the number of reflectors when reflector candidate locations and obstacles are fixed, while the number of mmWave logical links varies. In this set of experiments, all logical links are feasible. More reflectors are needed as the number of logical links increases. However, the relationship is not always linear due to the absence of LOS paths between TX/RX pairs and the multiplexing of reflectors.

Fig. 3(b) show the percentage of blockage time per link when human subjects move randomly in the room. The mobility setup is similar to that in Section III-B. Clearly, As the number of human subjects increases, the percentage of blockage time increases as well. However, the robust scheme leads to 50% less blockage.

Next, we evaluate the impact of robustness index ρ . In this setup, 1 human subject moves randomly and there are 5 logical links. Figure 3(c) shows the number of reflectors used and the percentage of blockage time. As expected, as ρ increases, more reflectors are used and the blockage time reduces.

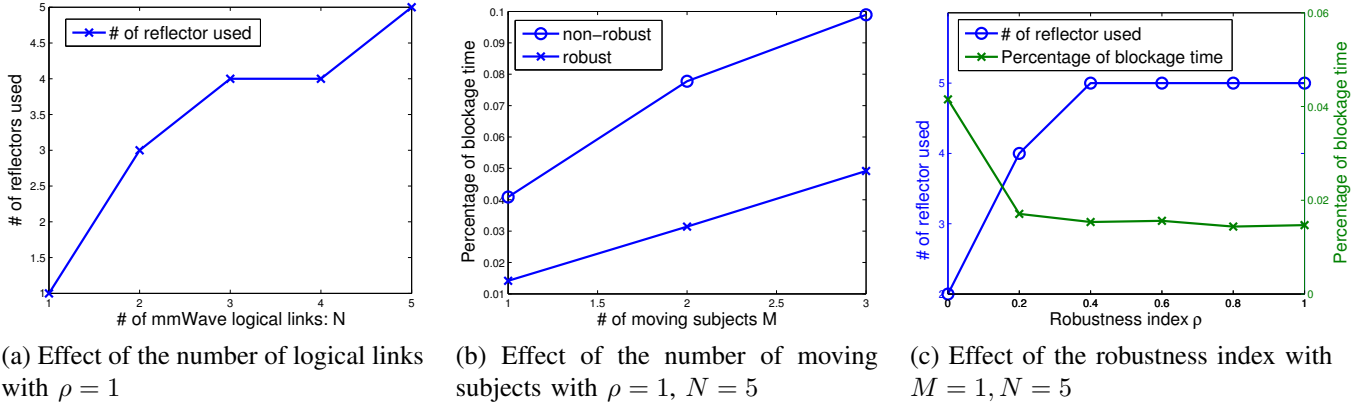


Fig. 3: Performance of RMRP in an mmWave home network deployed in a 10m×10m room.

B. RMURP Performance

We now evaluate the performance of the *Bisection Search* and *GBD* algorithms on RMURP. The error tolerance of *Bisection Search* is set to $TOL = 1.0$.

Fig. 4 shows the utility achieved using both methods by varying the number of logical links (Fig. 4(a)), the total number of reflectors (Fig. 4(b)) and the robustness index (Fig. 4(c)). In all cases, *GBD* achieves higher utility compared to the *Bisection Search* method. Reducing the threshold TOL improves the performance of the *Bisection Search* but comes at a higher computation cost.

Fig. 5(a) demonstrates the convergence of the *GBD* algorithm. As shown in Fig. 5(a), over time, the upper bound (solutions to the master problem) is non-increasing; and the lower bound (solutions to the primary problem) is non-decreasing. The algorithm converges to the optimal solution after 50 iterations when the upper bound equals to the lower bound.

Fig. 5(b)(c) shows the percentage of link blockage time per link under RMURP when human subjects move randomly in the room by varying the number of moving subjects and the robustness index, respectively. In both cases, *GBD* achieves lower percentage of blockage. This implies that the reflector selection in *GBD* has more spatial diversity.

VII. CONCLUSION

In this paper, we formulated robust reflector placement problems in mmWave WPANs, namely, the robust minimum reflector placement problem (RMRP) and robust maximum utility reflector placement (RMURP) for better connectivity and robustness against link blockage. Under the D-norm uncertainty model, RMRP and RMURP were casted as MILP and MINLP problems. Efficient algorithms were devised and evaluated using simulations.

For future work, we will incorporate more complex interference and antenna models in the robustness formulation. Another interesting agenda is to explore the use of passive reflectors in mmWave WPANs.

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APPENDIX

A. Proof of the NP-hardness of RMRP

Definition 4: We call the special case of RMRP problem, where the robustness index $\rho = 0$, the MRP problem.

Lemma A.1: MRP is NP-hard.

Proof: Since $\rho = 0 \Rightarrow \Gamma_k \equiv 0$ in MRP, the *protection function* of (13) is null, which means only primary paths are considered in (10). In MRP, r_{ik} represents the percentage of reflector k 's capacity occupied by logical link i 's primary path, if i choose to route its primary path via k . If we treat a bin as a reflector's capacity, and treat the volume of an item as the percentage of reflector capacity occupied by an logical link, the *Bin-Packing* problem can be reduced to MRP in a polynomial time. Since *Bin-Packing* is NP-hard [23], MRP is also a NP-hard problem. ■

As MRP is just a special case of RMRP, RMRP is henceforth harder than MRP, so RMRP is also NP-Hard.

B. Proof of the NP-hardness of RMURP

Definition 5: We call the special case of RMURP problem, where the mmWave network adopts robustness index of $\rho = 0$ and only has one candidate reflector location, as MURP problem.

Lemma A.2: MURP is NP-hard.

Proof: Since $\rho = 0 \Rightarrow \Gamma_k \equiv 0$ in MURP, the *protection function* term is null. Also, consider only one candidate reflector location k in the network, the scheduling constraint of MURP in (17) can be rewritten as: $\sum_i \eta_i x_{ik} \alpha r_i \tau_i \leq z_k$.

Let αr_i represent the value of item i , $\eta_i \alpha r_i \tau_i$ represent the weight of item i . the 0-1 knapsack problem can be reduced to MURP in a polynomial time. Since the *Knapsack* problem is NP-hard [23], MURP is NP-hard. ■

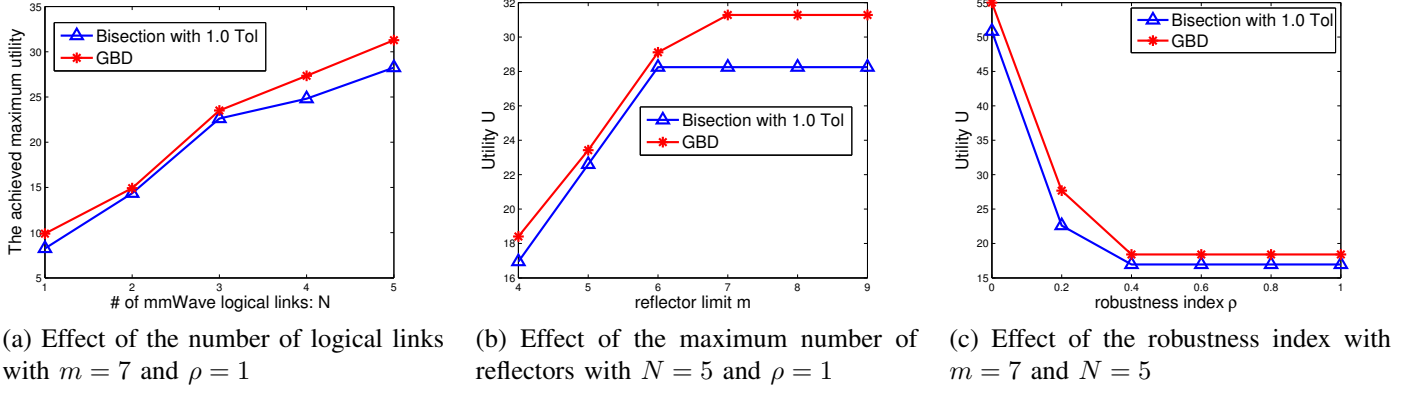


Fig. 4: Performance of two RMURP solutions in an mmWave home network deployed in a $10\text{m} \times 10\text{m}$ room, *Bisection Search* algorithm with 1.0 tolerance and *GBD* algorithm.

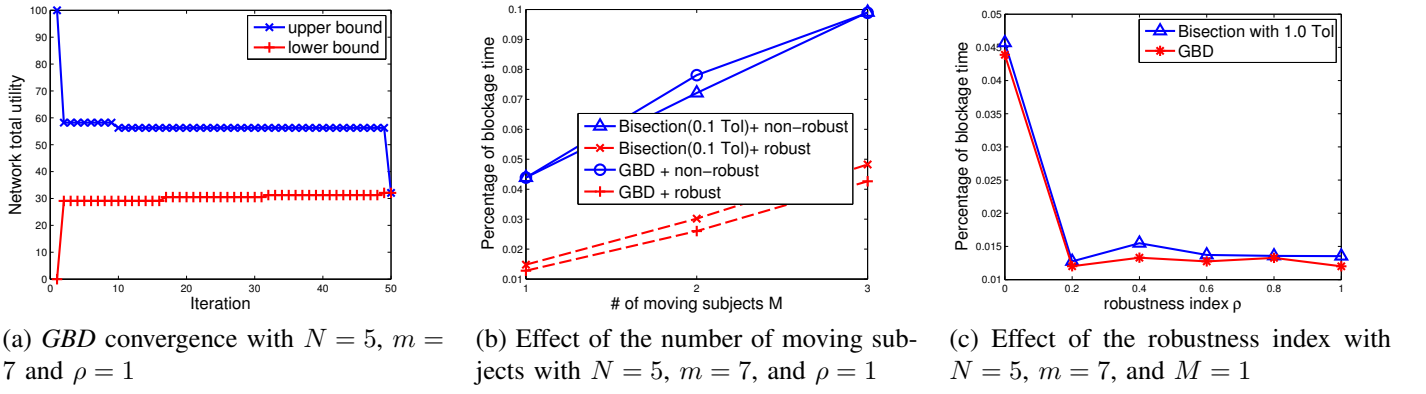


Fig. 5: Performance of RMURP in an mmWave home network deployed in a $10\text{m} \times 10\text{m}$ room.

As MURP is a special case of RMURP, RMURP is henceforth harder than MURP. This proves the lemma.

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